Ultra Reliable Low Latency Routing in LEO Satellite Constellations: A Stochastic Geometry Approach

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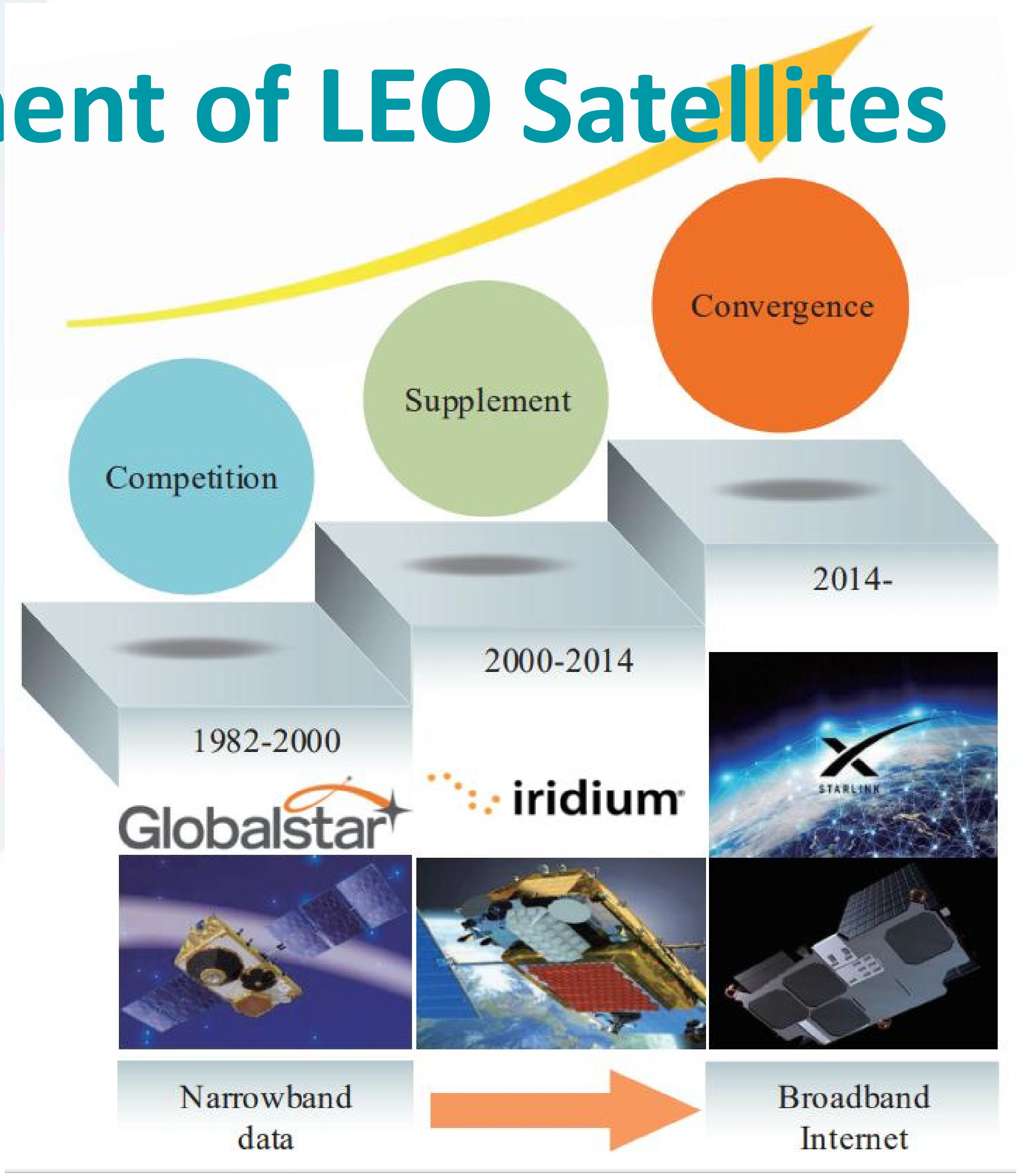
OUTLINE

1. Motivation
2. SG-Based Satellite Routing
3. System-Level Metrics
4. Optimization Problems
5. Performance Evluation for Algorithms
6. Numerical Simulation

Development of LEO Satellites



The state of the art

Graph theory model: based-on deterministic network topologies. Not suitable for dynamic networks.

Random algorithms: difficult to be supported by theoretical analysis, high complexity.

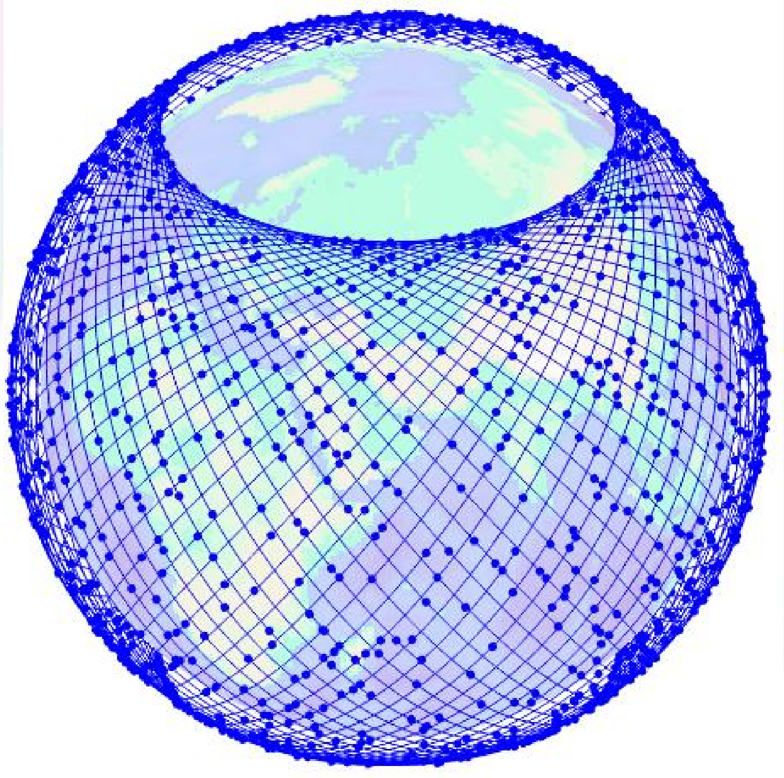
Fuction mapping

Stochastic Geometry

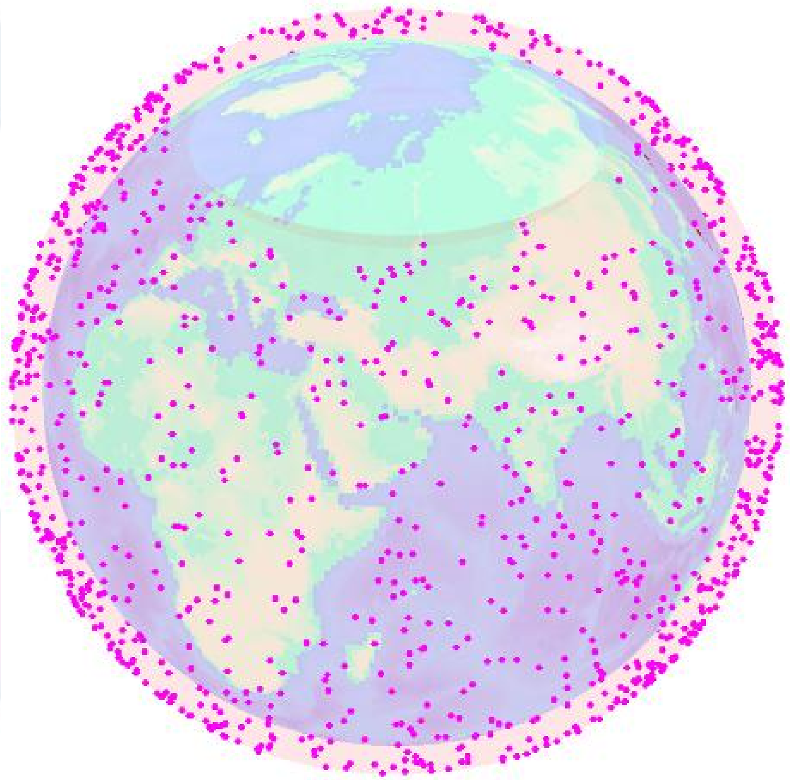
Stochastic Distribution

BPP Model

Tractable and accurate



Orbit geometry model



Binomial point process

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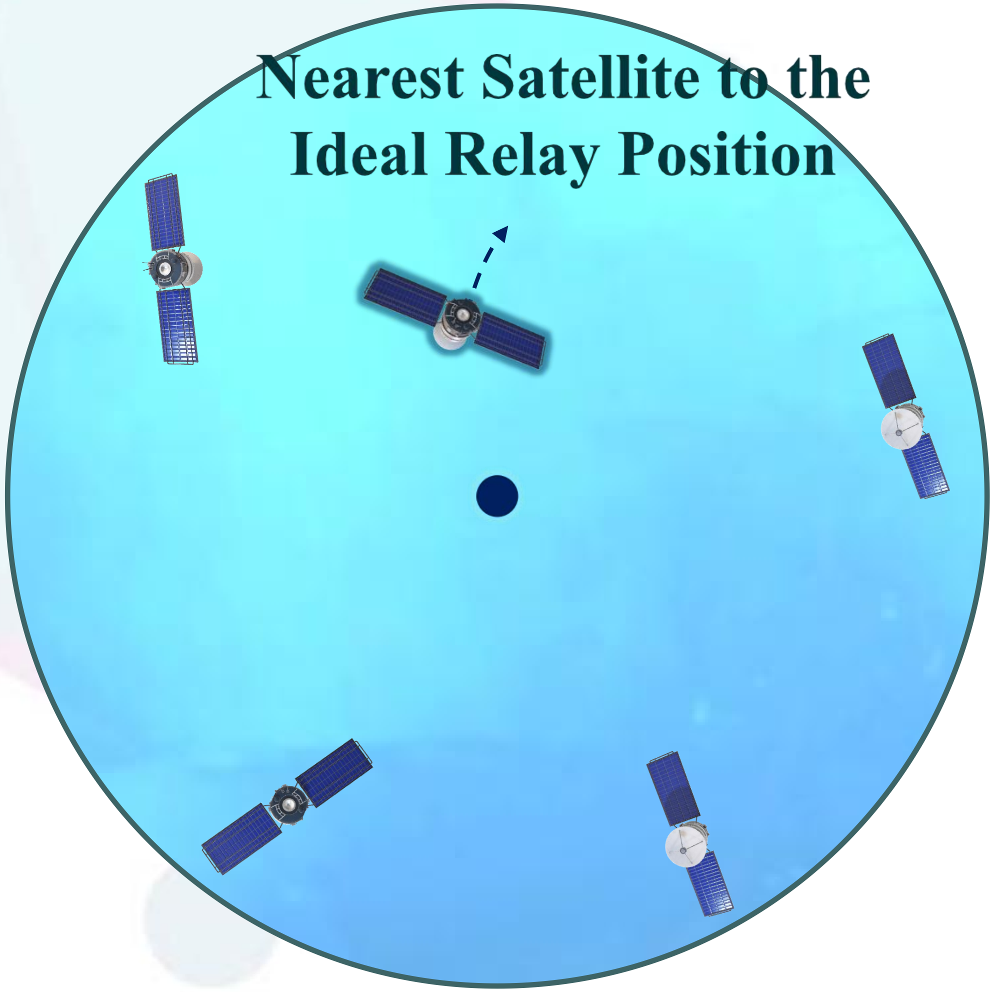
Step 1: Find the Shortest Inferior Arc

Step 2: Obtain Ideal Relay Positions

Step 3: Select the Nearest Satellite



Issues for Step 1

Issues for Step 2

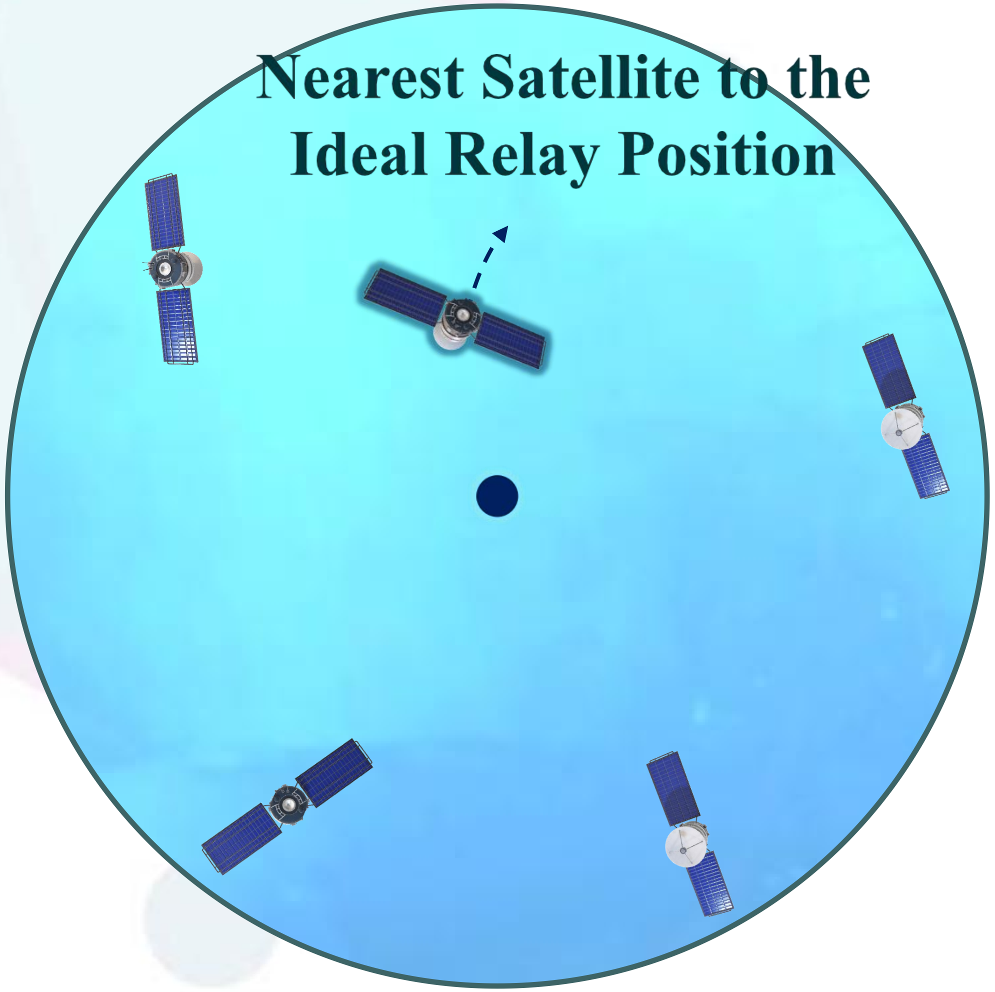
What is the optimal number of hops?

Depends on system-level metrics

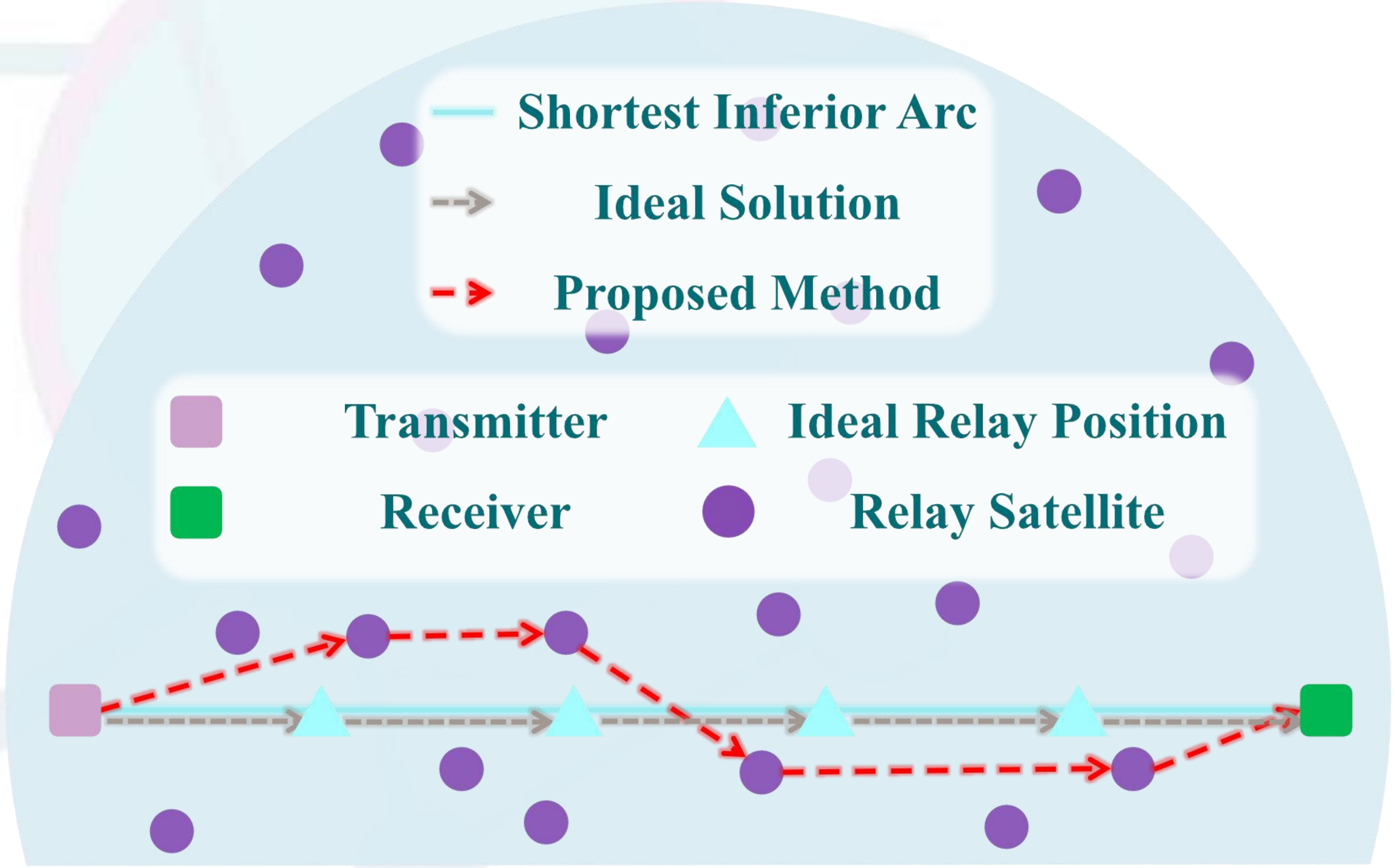
Where are optimal relay positions?

Proposition: Distributed **at equal intervals** for inter-satellite routing

Issues for Step 3



Distance scaling factor



Distance scaling factor

$$\alpha = \frac{\text{Average distance in BPP}}{\text{Average distance in ideal scenario}}$$

$$\alpha^{(1)}(\theta_i) = \frac{N_s}{8\pi R_s \sin\frac{\theta_i}{2}} \int_0^{2\pi}\int_0^{\pi} \sin\xi \left(\frac{1+\cos\xi}{2}\right)^{N_s-1} \times d(\theta_i, 0; \xi, \varphi)\, \mathrm{d}\xi \mathrm{d}\varphi, \quad i = \{1, N_l\},$$

$$\alpha^{(2)}(\theta_i) = 2\alpha^{(1)}(\theta_i) - 1 \ (\textit{called additive evaluation})$$

$$\alpha^{(2)}(\theta_i) = \left(\alpha^{(1)}(\theta_i)\right)^2 \ (\textit{called multiplicative evaluation}).$$

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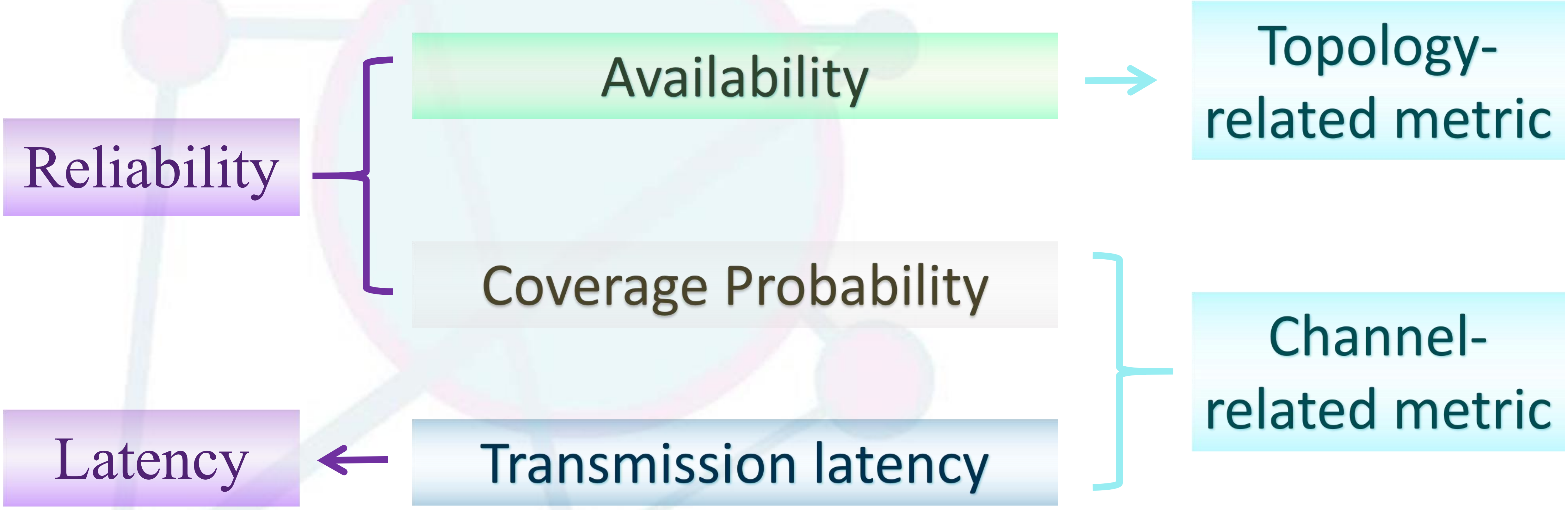
Issues for Step 2

What is the optimal number of hops?

Depends on system-level metrics

System-Level Metrics



Availability

Every relay satellite in the route is in the communication range (not blocked by the Earth) of its last hop and next hop

$$\underset{N_l, \mathcal{M}_{|N_l}}{\text{maximize}} \quad \prod_{i=1}^{N_l} \mathbb{1}\left\{ l_i \leq 2\sqrt{R_s^2 - R_\oplus^2} \right\}$$

System-Level Metrics

Coverage Probability

Transmission latency

Channel-related metric

Channel Model

Large Scale Fading

Small Scale Fading

$$S = G\left(\frac{\lambda}{4\pi}\right)^2 \rho W r^{-2}$$

Transmit Power

Path Loss

Small scale fading

W → **Pointing Error**

$$f_{W|\theta_d}(w) = \frac{\eta_s^2 w^{\eta_s^2 - 1} \cos\theta_d}{A_0^{\eta_s^2}}, \quad 0 \leq w \leq A_0$$

$$f_{\theta_d}(\theta_d) = \frac{\theta_d}{\varsigma^2} \exp\left(-\frac{\theta_d^2}{2\varsigma^2}\right), \quad \theta_d \geq 0.$$

Channel-Related Metrics

$$\mathrm{SNR} = \frac{S}{\sigma^2}$$

SNR: The signal-to-noise ratio

$$P^C = \mathbb{P}[\mathrm{SNR} > \gamma]$$

Coverage Probability: the probability of successful signal demodulation at the receiver

$$\tau = \mathbb{E}\left[\frac{\varpi}{B\log_2(1+\mathrm{SNR})}\right]$$

Transmission latency: latency required for the data packet transmission at the achievable channel capacity

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Original Optimization Problem

$$\mathscr{P}_1 : \underset{N_l, \mathcal{M}_{|N_l}}{\text{maximize}} \quad \prod_{i=1}^{N_l} \mathbb{1}\left\{ l_i \leq 2\sqrt{R_s^2 - R_\oplus^2} \right\},$$

$$\underset{N_l, \mathcal{M}_{|N_l}}{\text{maximize}} \quad \prod_{i=1}^{N_l} P_{\text{cond}}^{C}(l_i),$$

$$\underset{N_l, \mathcal{M}_{|N_l}}{\text{minimize}} \quad \sum_{i=1}^{N_l} \frac{\varpi}{B \log_2(1 + \text{SNR}_i)},$$

$$P_{\text{cond}}^{C}(l_i) = \mathbb{P}\left[\text{SNR}_i = \frac{\rho_i^r}{\sigma^2} \geq \gamma | \, l_i \right]$$

Method 1 : Optimization with Constrains

$$\mathscr{P}_2 : \underset{N_l}{\text{minimize}} \; \underset{\mathcal{M}_{|N_l}}{\text{minimize}} \quad \sum_{i=1}^{N_l} \frac{\varpi}{B \log_2 \left(1 + \mathbb{E}_W [\mathrm{SNR}_i]\right)}, \tag{9a}$$

$$\text{subject to} \quad \prod_{i=1}^{N_l} P_{\text{cond}}^{C}(l_i) \geq 1 - \varepsilon, \tag{9b}$$

$$l_i \leq 2\sqrt{R_s^2 - R_{\oplus}^2}, \; \forall i, \tag{9c}$$

$$P_{\text{cond}}^{C}(l_i) = 1 - \varsigma^2 - \left(1 - \varsigma^2\right) \left(\frac{\gamma \sigma^2 \left(4\pi l_i\right)^2}{A_0 \rho^t G \lambda^2} \right)^{\eta_s^2}$$

Method 2 : Optimization Through Mutual Relations

$$\mathscr{P}_3 : \underset{\mathcal{M}_{|N_l}}{\text{minimize}} \ \underset{N_l}{\text{minimize}} \quad T_{\text{ARQ}} = \sum_{i=1}^{N_l} \frac{1}{P_i^{A,C}} \left(\frac{\varpi}{B \log_2 (1 + \text{SNR}_i)} \right)$$

Automatic repeat request (ARQ) protocol: the receiver respond with ”successful” when packet is received; the transmitter sends the packet repeatedly without waiting until ”successful” is received.

$P_i^{A,C}$ is the probability that the i^{th} hop is available, and its received SNR is greater than the coverage threshold

Problem Solving

Input: Locations of satellites $\mathcal{X}$.
Obtain optimal number of hops N_l^* through Algorithm 1
for $i = 1 : N_l^* - 1$ **do**
  $m_i \leftarrow \arg \min_{1 \leq n \leq N_s} d\left(i \times \frac{\Theta}{N_l}, 0; \psi_n, \phi_n\right)$.
  $\mathcal{B}_i \leftarrow \mathbb{1}\left\{d\left(\psi_{m_{i-1}}, \phi_{m_{i-1}}; \psi_{m_i}, \phi_{m_i}\right) > 2\sqrt{R_s^2 - R_\oplus^2}\right\}$.
end for
$\mathcal{B}_{N_l^*} \leftarrow \mathbb{1}\left\{d\left(\psi_{m_{N_l^*-1}}, \phi_{m_{N_l^*-1}}; \Theta, 0\right) > 2\sqrt{R_s^2 - R_\oplus^2}\right\}$.
$\mathcal{M}_{|N_l^*} \leftarrow \{m_1, m_2, ..., m_{N_l^*-2}, m_{N_l^*-1}\}$.
Output: IDs of relay satellites in the route $\mathcal{M}_{|N_l^*}$

Input: Tolerable probability of communication interruption ε.
Initiate $N_l^* \leftarrow 0$ and $T_{\min} \leftarrow \infty$.
if $\frac{\Theta}{2\arccos(R_\oplus/R_s)} \geq \frac{\ln(1-\varepsilon)}{\ln(1-\varsigma^2)}$ **then**
  Exit the algorithm and **output** N_l^*.
end if
$N_l \leftarrow \left\lceil \frac{\Theta}{2\arccos(R_\oplus/R_s)} \right\rceil$.
while $\frac{\Theta}{2\arccos(R_\oplus/R_s)} \leq N_l < \frac{\ln(1-\varepsilon)}{\ln(1-\varsigma^2)}$ **do**
  $N_l \leftarrow N_l + 1$, $l_{\text{Alg1}} \leftarrow 2R_s \sin \frac{\Theta}{2N_l}$.
  if $\left(P_{\text{cond}}^C\left(\alpha^{(1)}\left(\frac{\Theta}{N_l}\right) l_{\text{Alg1}}\right)\right)^2 \times \left(P_{\text{cond}}^C\left(\alpha^{(2)}\left(\frac{\Theta}{N_l}\right) l_{\text{Alg1}}\right)\right)^{N_l-2} \geq 1 - \varepsilon$ and $\alpha^{(2)}\left(\frac{\Theta}{N_l}\right) l_{\text{Alg1}} < 2\sqrt{R_s^2 - R_\oplus^2}$ **then**
    $T_{\text{Alg1}} \leftarrow 2T_{\text{tx},1}^*\left(\alpha^{(1)}\left(\frac{\Theta}{N_l}\right) l_{\text{Alg1}}\right) + (N_l - 2)\, T_{\text{tx},1}^*\left(\alpha^{(2)}\left(\frac{\Theta}{N_l}\right) l_{\text{Alg1}}\right)$.
    if $T_{\min} \geq T_{\text{Alg1}}$ **then**
      $T_{\min} \leftarrow T_{\text{Alg1}}$, $N_l^* \leftarrow N_l$.
    end if
  end if
end while
Output: Optimal number of hops N_l^*.

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Availability - Double Integral

$$P^A = \left(F_{\theta_c^{(1)}} \left(2 \arcsin \frac{\sqrt{R_s^2 - R_\oplus^2}}{R_s} \right) \right)^2 \times \left(F_{\theta_c^{(2)}} \left(2 \arcsin \frac{\sqrt{R_s^2 - R_\oplus^2}}{R_s} \right) \right)^{N_l - 2}$$

$$F_{\theta_c^{(1)}} (\Xi) = \int_0^{2\pi} \int_0^{\Xi} \frac{N_s \sin \theta}{2^{N_s+1} \pi} \left(1 + \cos \left(2 \arcsin \frac{d(\theta, \phi; \frac{\Theta}{N_l}, 0)}{2 R_s} \right) \right)^{N_s - 1} \mathrm{d}\theta \mathrm{d}\phi,$$

$$F_{\theta_c^{(2)}} (\Xi) = F_{\theta_c^{(1)}} \left(2 \arcsin \frac{\sin(\Xi/2)}{\alpha^{(1)} (\Theta/N_l)} \right)$$

Coverage Probability - Double Integral

$$P^C_{\mathrm{rout}} = \left(\int_0^{\pi} f_{\theta_c^{(1)}}(\theta) P^C_{\mathrm{cond}}\left(2R_s \sin\frac{\theta}{2}\right) \mathrm{d}\theta\right)^2 \times \left(\int_0^{\pi} f_{\theta_c^{(2)}}(\theta) P^C_{\mathrm{cond}}\left(2R_s \sin\frac{\theta}{2}\right) \mathrm{d}\theta\right)^{N_l-2}$$

$$f_{\theta_c^{(1)}}(\theta) = \int_0^{2\pi} \frac{N_s \sin\theta}{2^{N_s+1}\pi} \left(1 + \cos\left(2\arcsin\frac{d(\theta,\phi;\frac{\Theta}{N_l},0)}{2R_s}\right)\right)^{N_s-1} \mathrm{d}\phi$$

$$f_{\theta_c^{(2)}}(\theta) = f_{\theta_c^{(1)}}\left(2\arcsin\frac{\sin(\theta/2)}{\alpha^{(1)}(\Theta/N_l)}\right) \frac{\cos(\theta/2)}{\sqrt{(\alpha^{(1)}(\Theta/N_l))^2 - \sin^2(\theta/2)}}$$

Transmission Latency - Triple Integral

$$\overline{T}_{\text{tx}} = 2\int_0^{\pi}\int_0^{A_0}\int_0^{\infty} \varpi f_{\theta_c^{(1)}}(\Xi) f_{W|\theta_d}(w) f_{\theta_d}(\theta_d) B^{-1} \log_2\left(1+\rho^t\, G\left(\frac{\lambda}{8\pi R_s \sin\frac{\Xi}{2}}\right)^2 \sigma^{-2} w\right)^{-1} \mathrm{d}\theta_d\,\mathrm{d}w\,\mathrm{d}\Xi$$

$$+ (N_l-2)\int_0^{\pi}\int_0^{A_0}\int_0^{\infty} \varpi f_{\theta_c^{(2)}}(\Xi) f_{W|\theta_d}(w) f_{\theta_d}(\theta_d) B^{-1} \log_2\left(1+\rho^t\, G\left(\frac{\lambda}{8\pi R_s \sin\frac{\Xi}{2}}\right)^2 \sigma^{-2} w\right)^{-1} \mathrm{d}\theta_d\,\mathrm{d}w\,\mathrm{d}\Xi,$$

Computationally complex

Approximate Transmission Latency - Single Integral

$$\widetilde{T}_{\mathrm{tx}} = 2\int_0^{\pi} f_{\theta_c^{(1)}}(\theta) T^*_{\mathrm{tx},1}\left(2R_s \sin\frac{\theta}{2}\right) \mathrm{d}\theta + (N_l - 2)\int_0^{\pi} f_{\theta_c^{(2)}}(\theta) T^*_{\mathrm{tx},1}\left(2R_s \sin\frac{\theta}{2}\right) \mathrm{d}\theta$$

$$T^*_{\mathrm{tx},1}(l_i) = \frac{\varpi}{B\log_2\left(1 + \rho^t G\left(\frac{\lambda}{4\pi l_i}\right)^2 \frac{A_0\eta_s^2}{1+\eta_s^2}\left(1-\varsigma^2\right)\sigma^{-2}\right)}$$

- We approximate it by considering the case where equality holds in Jensen's inequality
- We prove that Jensen's gap decreases as the SNR increases

ARQ Latency

$$\overline{T}_{\text{ARQ}} = 2\int_0^{A_0}\int_0^{\infty}\int_0^{\theta_{\max}} \frac{\varpi f_{\theta_c^{(1)}}(\Xi) f_{W|\theta_d}(w) f_{\theta_d}(\theta_d)}{P_{\text{cond}}^C\left(2R_s \sin\frac{\Xi}{2}\right) B \log_2\left(1+\rho^t G\left(\frac{\lambda}{8\pi R_s \sin\frac{\Xi}{2}}\right)^2 \sigma^{-2} w\right)} \mathrm{d}\Xi\, \mathrm{d}\theta_d\, \mathrm{d}w$$

$$+ (N_l - 2)\int_0^{A_0}\int_0^{\infty}\int_0^{\theta_{\max}} \frac{\varpi f_{\theta_c^{(2)}}(\Xi) f_{W|\theta_d}(w) f_{\theta_d}(\theta_d)}{P_{\text{cond}}^C\left(2R_s \sin\frac{\Xi}{2}\right) B \log_2\left(1+\rho^t G\left(\frac{\lambda}{8\pi R_s \sin\frac{\Xi}{2}}\right)^2 \sigma^{-2} w\right)} \mathrm{d}\Xi\, \mathrm{d}\theta_d\, \mathrm{d}w$$

Approximate

$$\widetilde{T}_{\text{ARQ}} = 2\int_0^{\theta_{\max}} f_{\theta_c^{(1)}}(\theta) \frac{T^*_{\text{tx},1}\left(2R_s \sin\frac{\theta}{2}\right)}{P_{\text{cond}}^C\left(2R_s \sin\frac{\theta}{2}\right)} \mathrm{d}\theta + (N_l - 2)\int_0^{\theta_{\max}} f_{\theta_c^{(2)}}(\theta) \frac{T^*_{\text{tx},1}\left(2R_s \sin\frac{\theta}{2}\right)}{P_{\text{cond}}^C\left(2R_s \sin\frac{\theta}{2}\right)} \mathrm{d}\theta.$$

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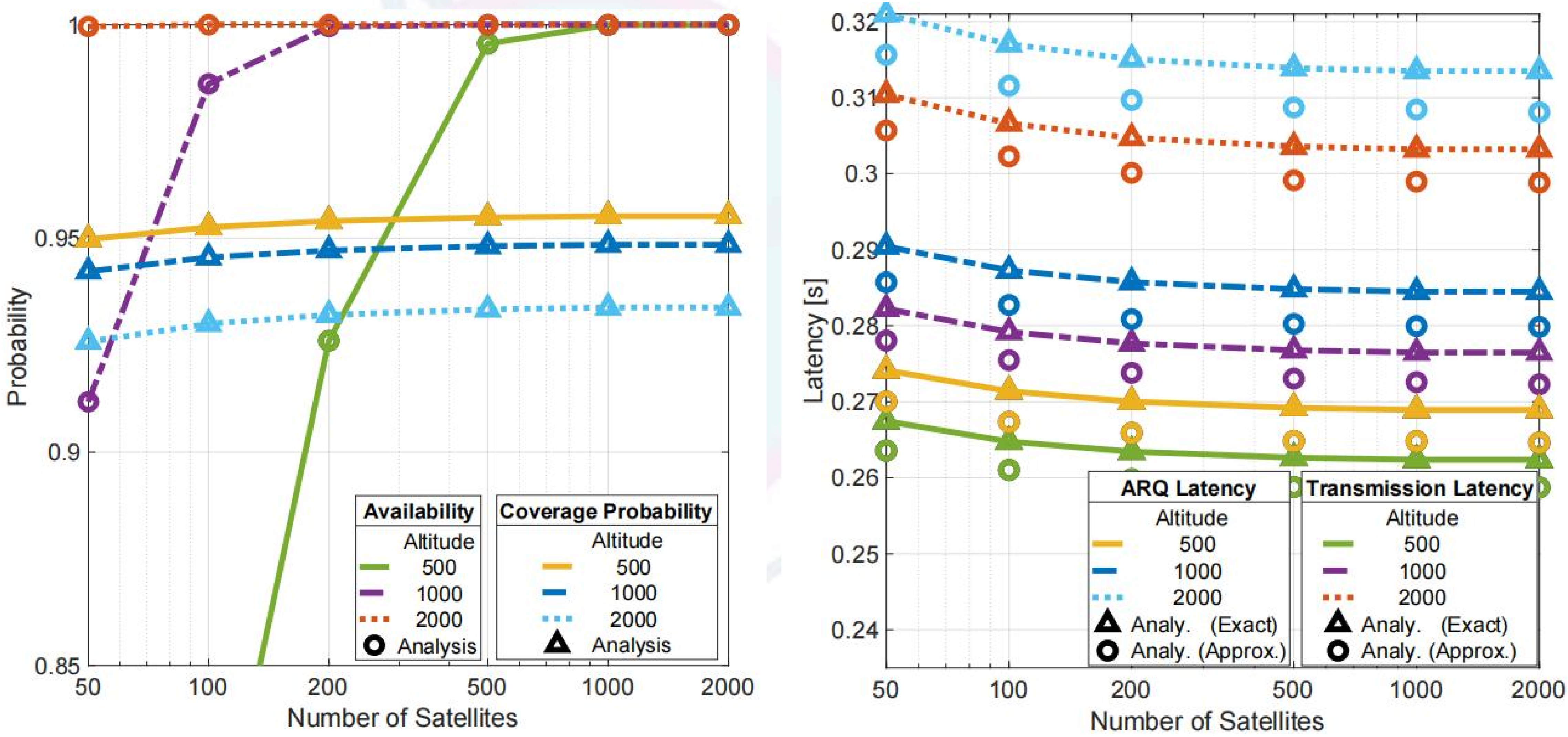
Results of Deterministic Constellations

Constellation	Starlink	Kuiper	OneWeb
Number of satellites	11927	3236	648
Altitude [km]	550	610	1200
Interference $\times 10^{-16}$ [mw]	27.81	5.359	0.908
Noise [mw]	10^{-10}		
Optimal number of hops (method I)	5	5	6
Optimal number of hops (method II)	5	5	6
Availability (Simu. & Analy.)	1.000	1.000	1.000
Coverage probability (Simu. & Analy.)	0.909	0.908	0.907
Transmission latency [s] (Simu. & Analy.)	0.642	0.645	0.741
Approx. transmission latency [s] (Analy.)	0.628	0.632	0.729
Propagation latency [s] (Simu.)	0.071	0.072	0.079
ARQ latency [s] (Simu. & Analy.)	0.654	0.658	0.754
Approx. ARQ latency [s] (Analy.)	0.641	0.645	0.741

Constellation Configuration Design



Strategies Comparison

Ideal Solution

Proposed Strategy

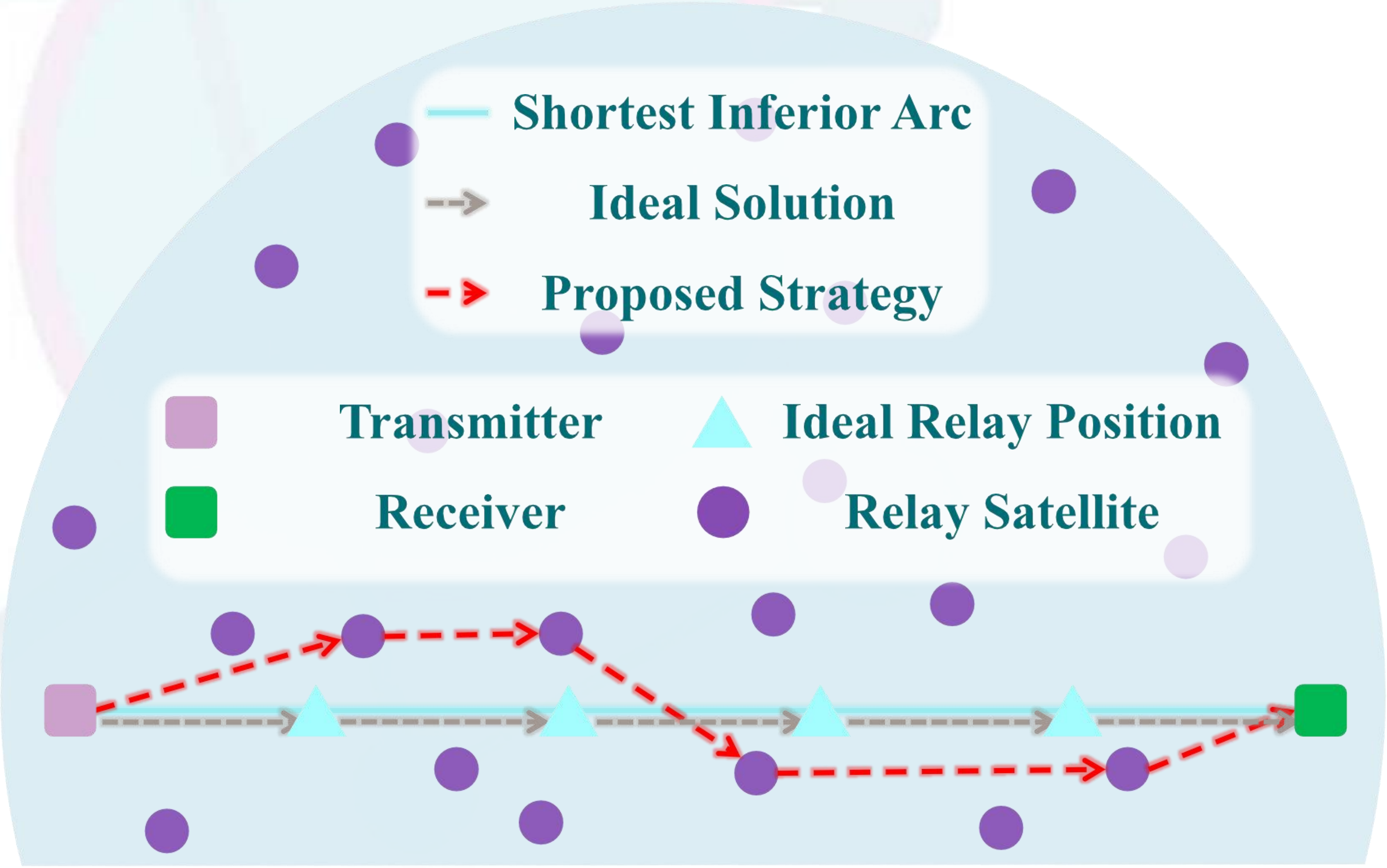
Minimum Deflection Angle

Maximum Stepsize (Greedy)

Minimum Stepsize

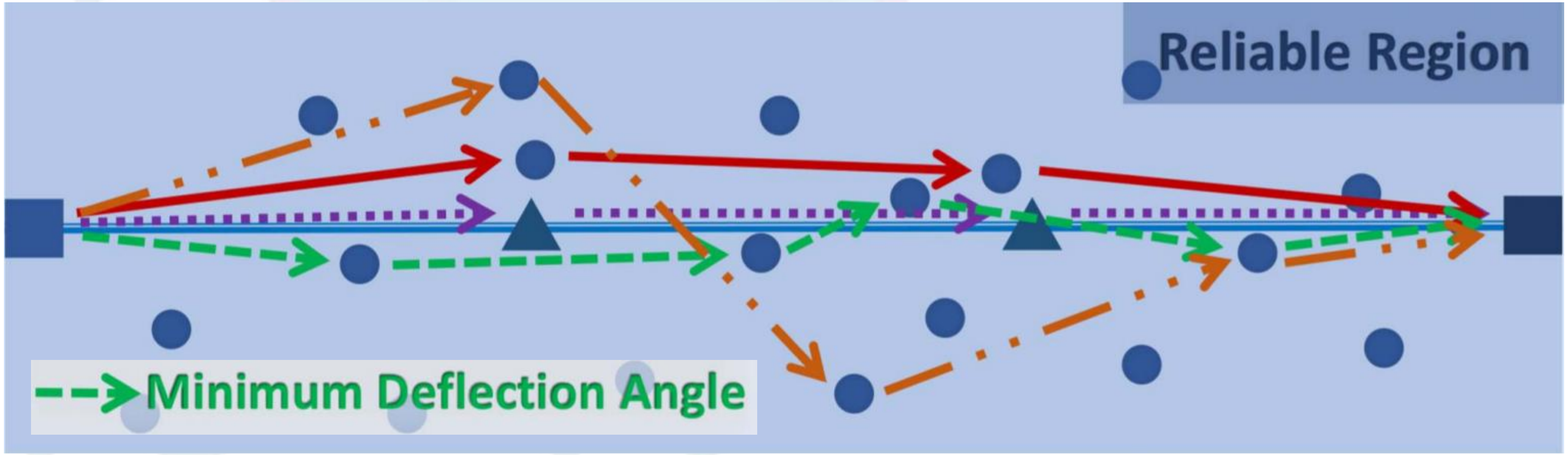
SG-Based Routing Strategies

Ideal Solution



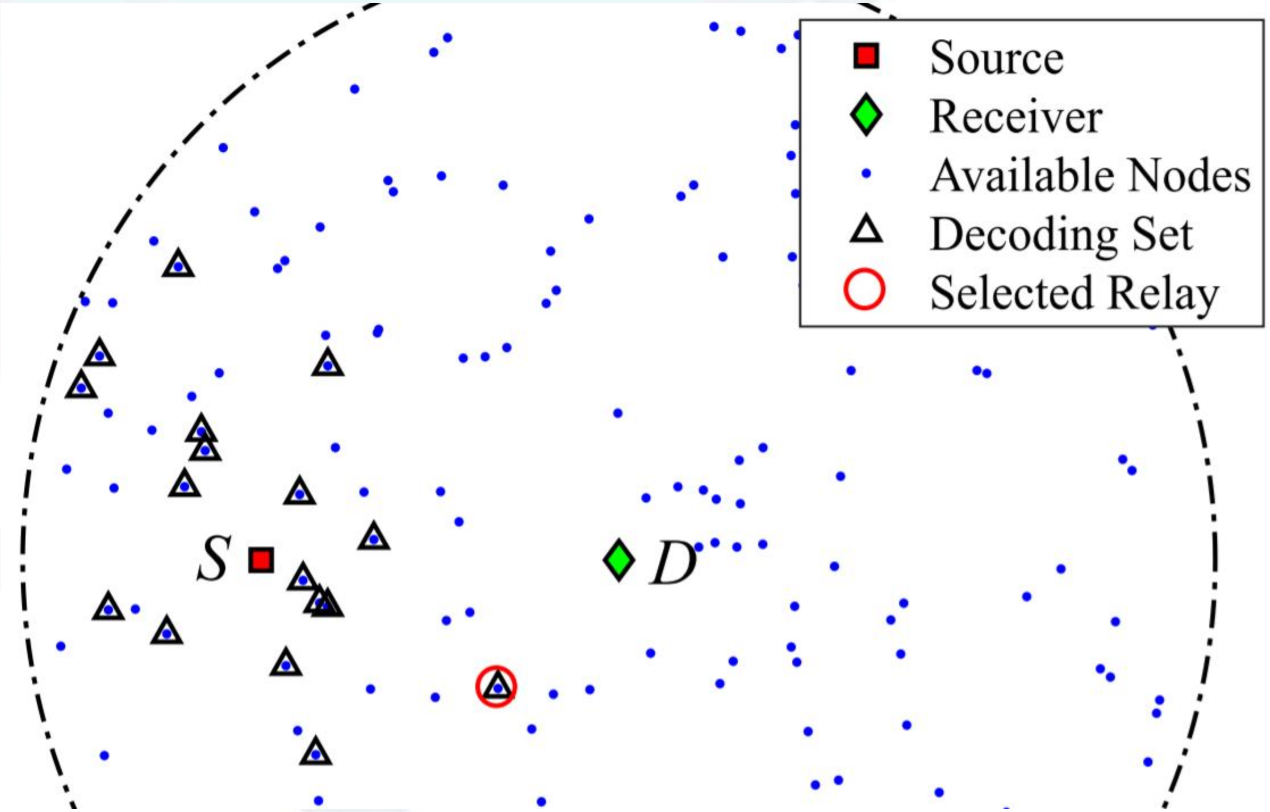
SG-Based Routing Strategies

Minimum Deflection Angle



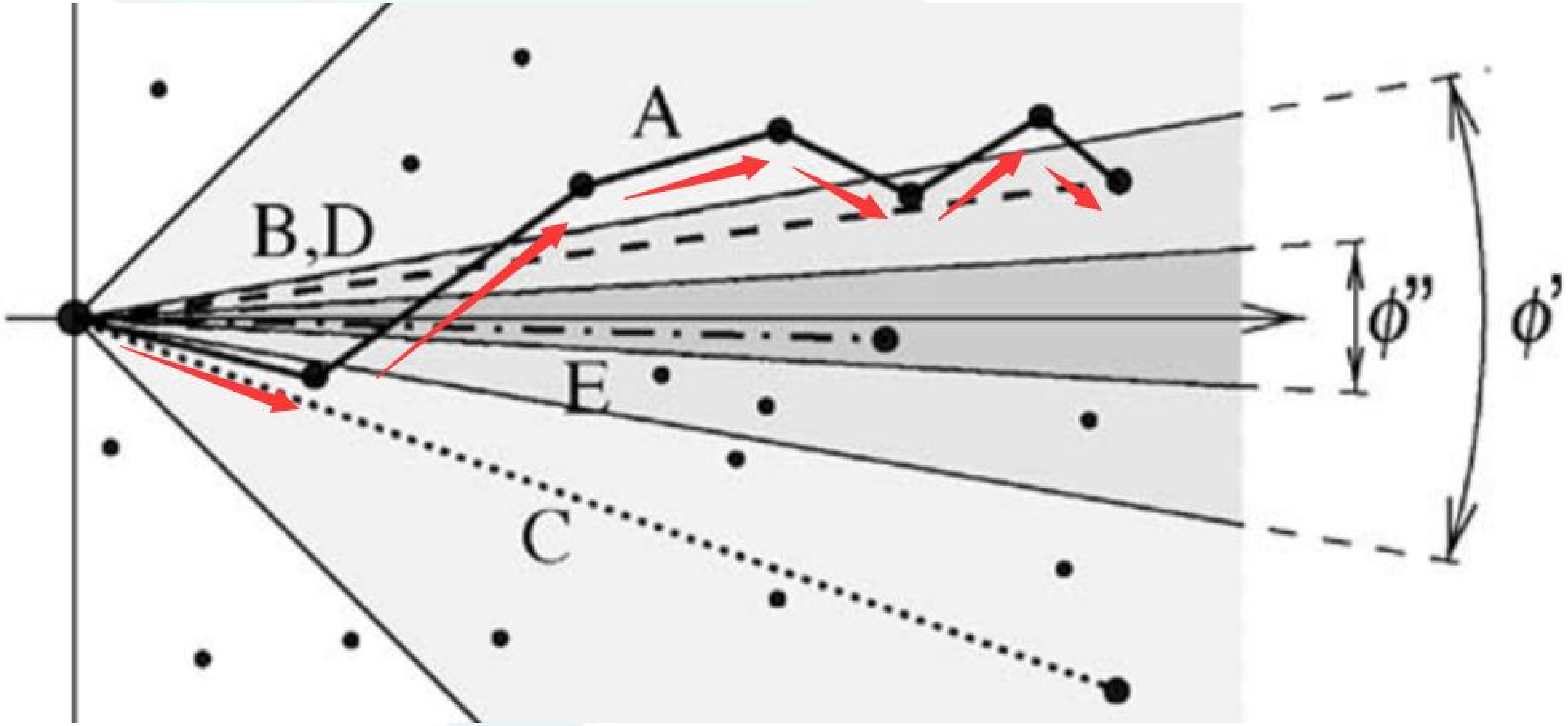
SG-Based Routing Strategies

Maximum Stepsize (Greedy)

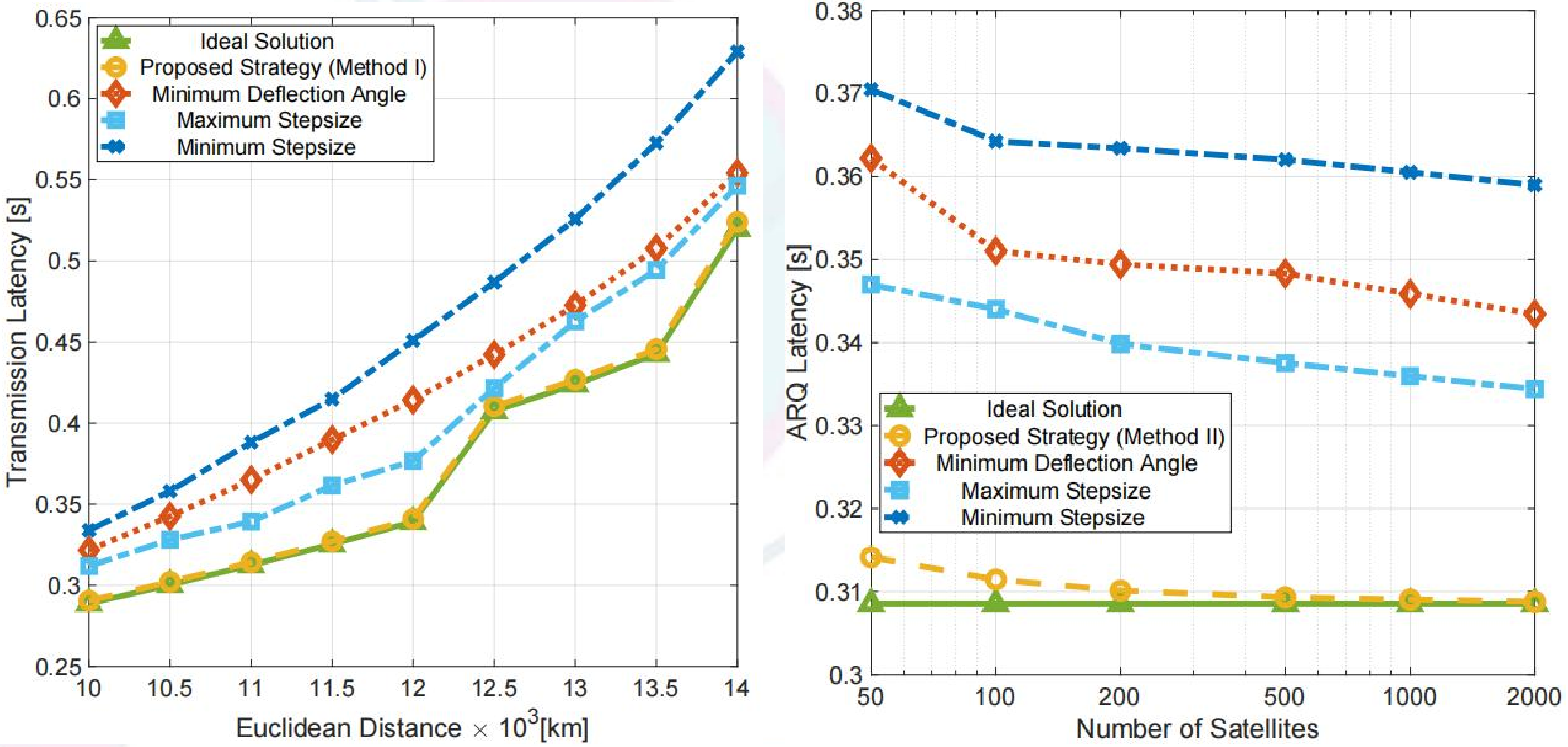


SG-Based Routing Strategies

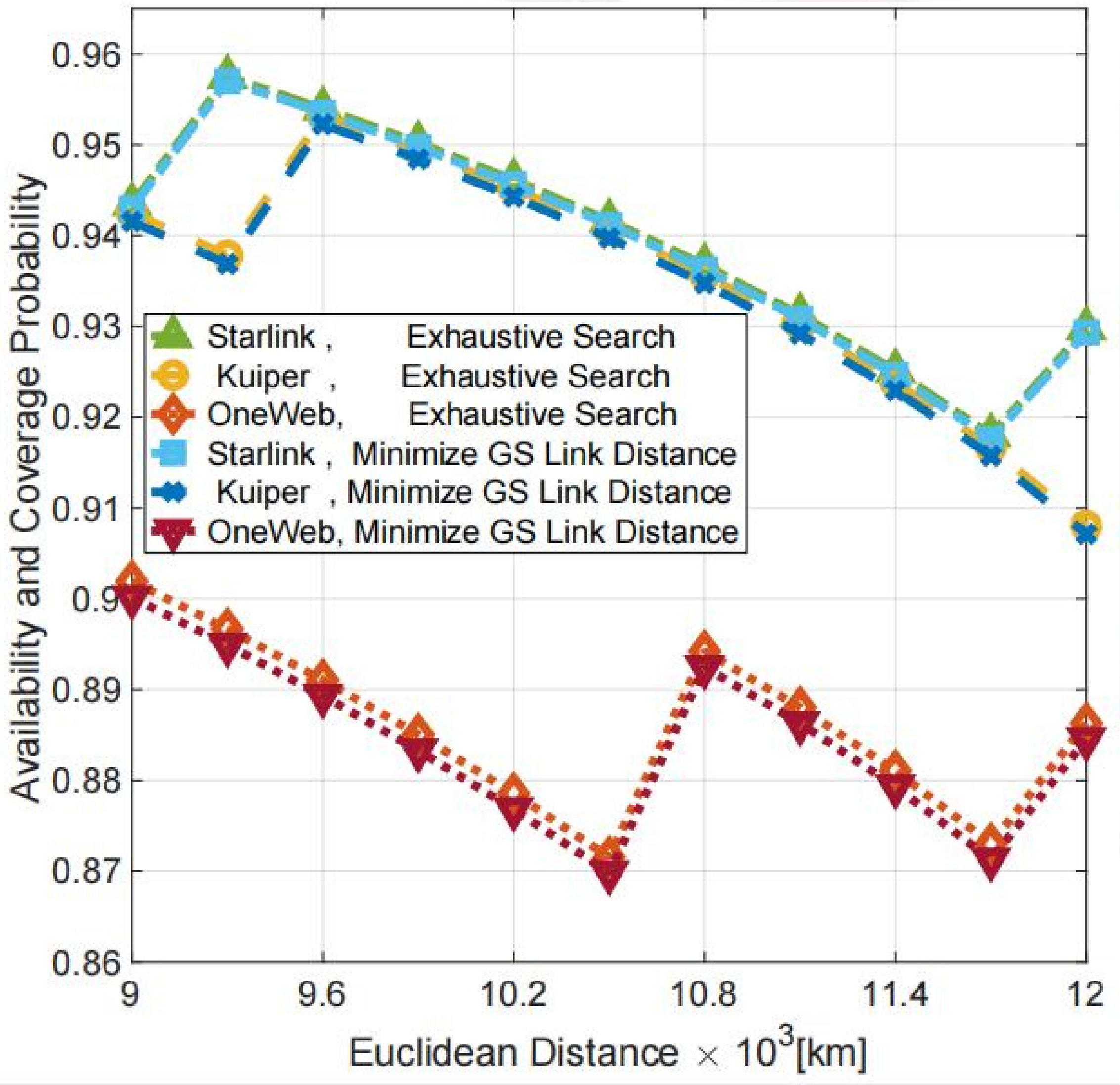
Minimum Stepsize

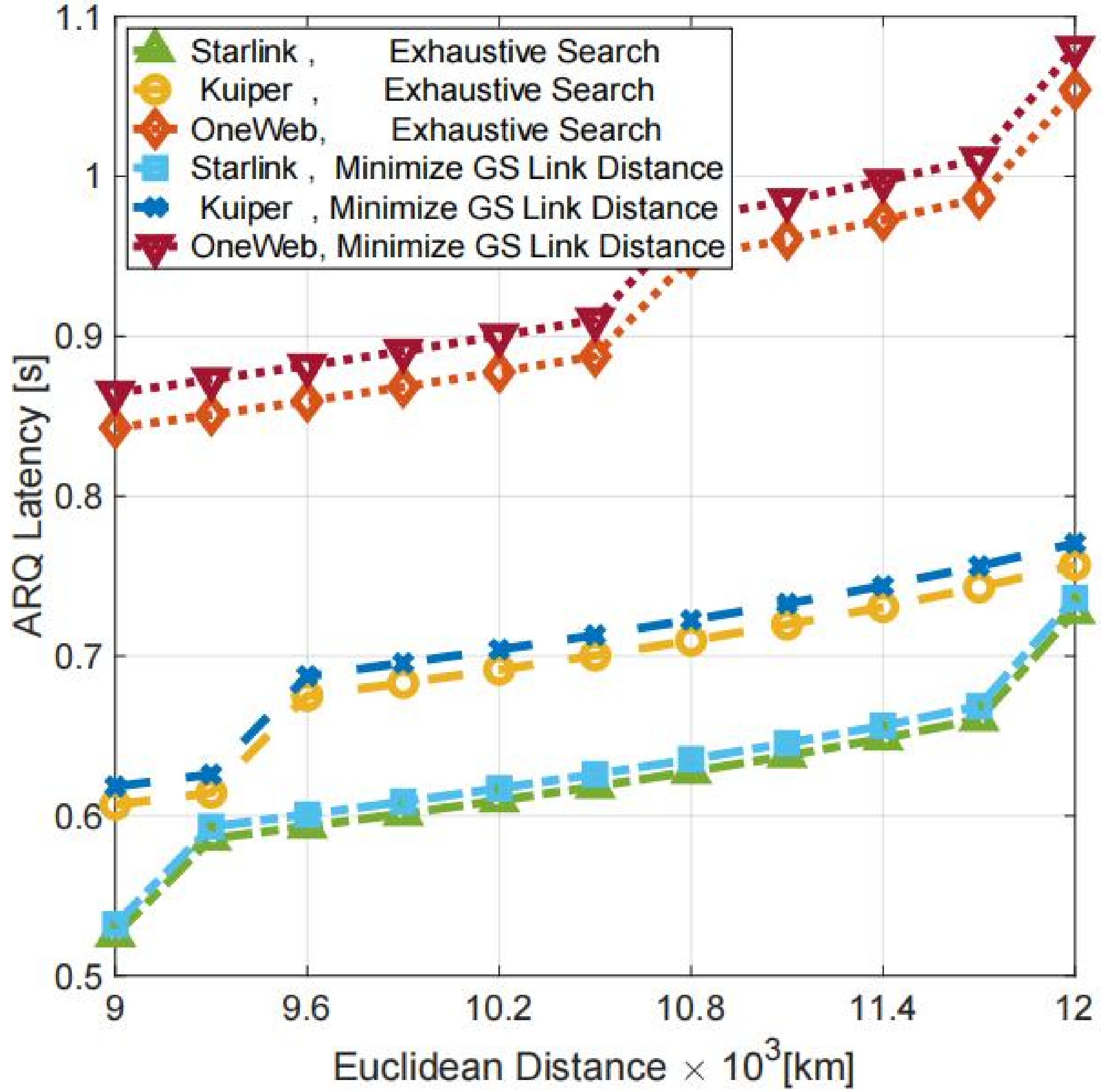


Strategies Comparison



Extension to Satellite-Terrestrial Communication





Thank You !